Test 1

Differentiaton , applications and Optimisation.

Basic antidifferentiation

Semester One 2018

Year 12 Mathematics Methods

Calculator Free

PERTH MODERN SCHOOL

Exceptional schooling. Exceptional students.

Name: CHENG

Teacher:

_______ Mr McClelland

_______ Mrs. Carter

_______ Mr Gannon

_______ Ms Cheng

_______ Mr Staffe

_______ Mr Strain

Date Monday 20th February 7.45am

You may have a formula sheet for this section of the test.

Total________/21 **20 Minutes**

Question 1 **(3 marks)**

Given that the function f has a rule of the form $f(x) = ax^2 + bx$ and $f(1) = 6$ and $f'(1) = 0$, find the values of a and b.

$f(1) = a + b = 6$ ①

$f'(1) = 2ax + b = 2a + b = 0$ ② ✓

$a = -6$ ✓

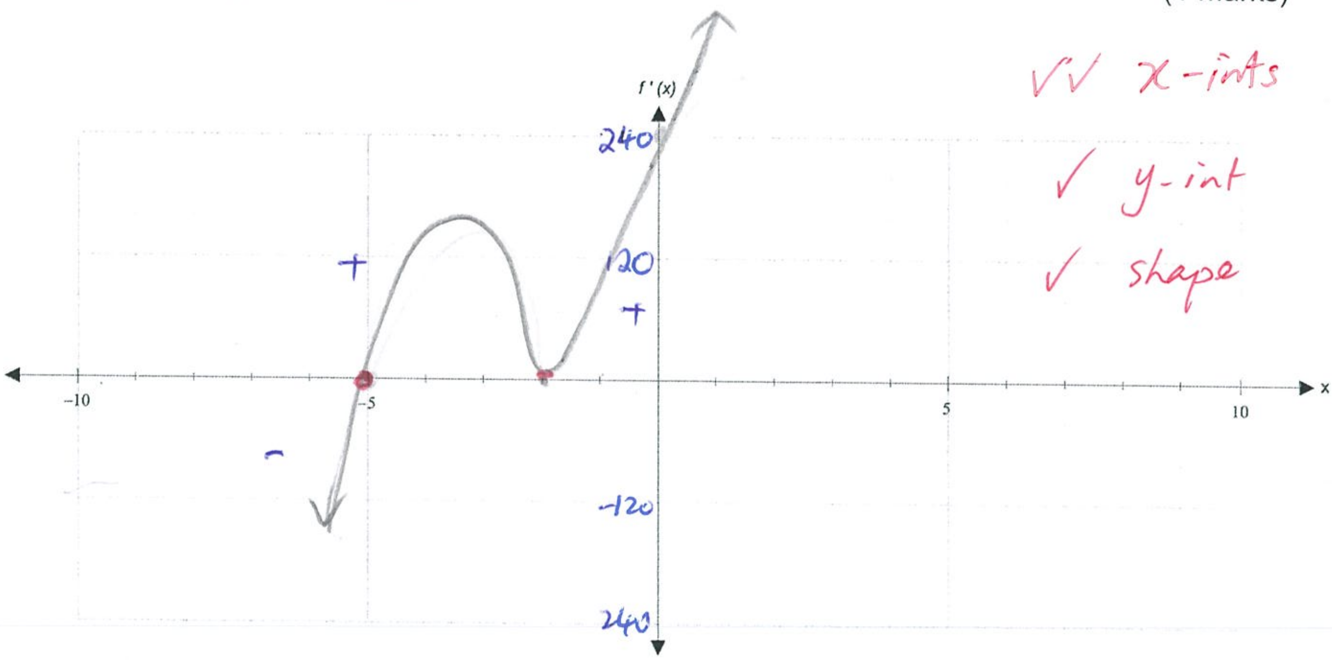
$b = 12$ ✓

Question 2 **(8 marks)**

Consider the gradient function $f'(x) = 12(x+2)^2(x+5)$.

$= 12(2)^2(5) = 240$

(a) Graph the gradient function (4 marks)



✓✓ x-ints
✓ y-int
✓ shape

(b) What kind of feature is at the point $(-5, -225)$ on the graph of $f(x)$? (2 marks)

$(-\infty, -5)$ $(-5, -2)$

$-$ $+$

$\downarrow$ $\uparrow$ $\therefore$ local min. ✓

T.P

min. T.P

✓T.P

(c) What kind of feature is at the point $(-2, -144)$ on the graph of $f(x)$? (2 marks)

$(-5, -2)$ $(-2, +\infty)$

$+$ $+$ ✓ Horizontal

$\therefore$ point of inflection. ✓

Question 3 **(6 marks)**

Clearly showing your use of the product, quotient or chain rule differentiate the following.

a) $10p(1-p)^9$ (2 marks)

$\frac{dy}{dx} = 10(1-p)^9 + 10p \times 9(1-p)^8 \times (-1)$

$\frac{dy}{dx} = 10(1-p)^9 - 90p(1-p)^8$

$\underline{-90p(1-p)^8}$

1 mark

b) $\frac{1}{\sqrt{x+2}}$ (~~2~~ 1 marks)

$= (x+2)^{-\frac{1}{2}}$

$\frac{dy}{dx} = -\frac{1}{2}(x+2)^{-\frac{3}{2}}$ $= \frac{1}{-2(x+2)^{3/2}}$

$= \frac{1}{-2\sqrt{(x+2)^3}}$

c) Consider the function $f(x) = (x-1)^2(x-2) + 1$ (~~2~~ 3 marks)

If $f'(x) = (x-1)(ux+v)$, where u and v are constants, use calculus to find the values of u and v.

$f'(x) = 2(x-1)(x-2) + (x-1)^2$
$= (x-1)(2x-4) + (x-1)(x-1)$
$= (x-1)(2x-4+x-1)$

$= (x-1)(3x-5) = 3x^2 - 8x + 5$

$\therefore \underline{u=3}, \underline{V=-5}$

$\frac{\pi}{\sqrt{\frac{l}{g}}}$ ✗ 1 mark

Question 4 **(4 marks)**

The time T seconds, for one complete swing of a pendulum of length $l\ m$, is given by the rule $T = 2\pi\sqrt{\frac{l}{g}}$, where g is a constant.

(a) Determine $\frac{dT}{dl}$, (2 marks)

$T = 2\pi\left(\frac{l}{g}\right)^{\frac{1}{2}} = 2\pi g^{-\frac{1}{2}} l^{\frac{1}{2}}$ ✓

$\frac{dT}{dl} = \frac{1}{2} \times 2\pi \times \left(\frac{l}{g}\right)^{-\frac{1}{2}} \times \left(\frac{1}{g}\right)$ ✓✓ $= \frac{1}{2} \times 2\pi \times g^{-\frac{1}{2}} l^{-\frac{1}{2}}$ ✓

$= \pi \times \frac{\sqrt{g}}{\sqrt{l}} \times \frac{1}{g} = \frac{\pi}{\sqrt{gl}} = \frac{\pi}{\sqrt{gl}}$

(b) Using the formula $\partial T \approx \frac{dT}{dl} \times \partial l$, find the approximate increase in T when l is increased from 1.6 to 1.7. Give the answer in terms of g. (2 marks)

$\delta T = \frac{\pi}{\sqrt{1.6g}}$ ✓ $\times 0.1$ ✓ sec

$= \frac{\pi}{\sqrt{0.16} \times \sqrt{10} \times \sqrt{g}} \times \frac{1}{10}$

$= \frac{\pi \times \sqrt{10}}{0.4 \times 10 \times \sqrt{g} \times 10}$

$= \frac{\pi\sqrt{10}}{40\sqrt{g}}$

$= \frac{\pi}{40}\sqrt{\frac{10}{g}}$ sec.

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Date th February 7.45am

You may have

- **a formula sheet**
- **one page of A4 notes, one side**
- **a scientific calculator**
- **a classpad**

Total__________/24 **25 minutes**

Question 1 **(9 marks)**

A model train travels on a straight track such that its acceleration after t seconds is given by $a(t) = pt - 13cm/s^2$, $0 \leq t \leq 10$, where p is a constant.

(a) Determine the initial acceleration of the model train. (1 mark)

$a(0) = p - 13 \ cm/s^2$ ✓

The model train has an initial velocity of $5cm/s$. After 2 seconds, it has a displacement of $-50cm$. A further 4 seconds later its displacement is 178 cm. i.e $t=6$

(b) Determine the value of the constant p. (4 marks)

$V(t) = \frac{pt^2}{2} - 13t + C \Rightarrow V(0) = C = 5$ ✓

$d(t) = \frac{pt^3}{6} - \frac{13t^2}{2} + 5t + d$ ✓

$d(t=2) = \frac{8p}{6} - \frac{13\times4}{2} + 10 + d = -50$

$d(t=6) = \frac{p\times6^3}{6} - \frac{13\times36}{2} + 6\times5 + d = 178$ ✓ (substitution)

$\therefore \begin{cases} p=12 \\ d=-50 \end{cases}$ ✓

(c) When is the model train at rest? (when $V(t)=0$) (2 marks)

$V(t) = \frac{12t^2}{2} - 13t + 5 = 6t^2 - 13t + 5 = 0$ 1 mark

$(2t-1)(3t-5) = 0$

$\therefore t = \frac{1}{2}$ sec or $\frac{5}{3}$ sec ✓✓

If $V=0$

$p\frac{t^2}{2} - 13t + 5 = 0$

(d) How far did the model train travel during the 8th second? (2 marks)

$d(t=8) - d(t=7)$ ✓

$= \left(\frac{12\times(8)^3}{6} - \frac{13\times(8)^2}{2} + 5\times8 - 50\right) - \left(\frac{12\times7^3}{6} - \frac{13\times7^2}{2} + 5\times7 - 50\right)$

$= 245.5$ cm ✓

Question 2 **(6 marks)**

A beverage company has decided to release a new product. “Modmash” is to be sold in $375\, mL$ cans that are perfectly cylindrical. {Hint: $1mL = 1cm^3$}

(a) If the cans have a base radius of $x\ cm$ show that the surface area of the can, S, is given by: $S = 2\pi x^2 + \frac{750}{x}$. (2 marks)

$$\boxed{h = \frac{375}{\pi x^2}}$$ 1mark

$$S = 2\pi x^2 + 2\pi x \times h$$
$$= 2\pi x^2 + 2\pi x \times \frac{375}{\pi x^2} \ ✓$$
$$= 2\pi x^2 + \frac{750}{x} \ ✓$$

(b) Using calculus methods, and showing full reasoning and justification, find the dimensions of the can that will minimise its surface area (4 marks)

$$S(x) = 2\pi x^2 + 750 \times x^{-1}$$
$$S'(x) = 4\pi x - 750x^{-2} = 0 \ ✓$$
$$x \approx 3.90796\ cm. \ ✓$$

x	$(-\infty, 3.90796)$	$(3.90796, +\infty)$
$S'(x)$	$-$	$+$
$S(x)$	$\downarrow$	$\uparrow$ ✓

(Explain why min)

Hence, $S(x)$ reaches min at $x = 3.90796$ cm
$= 3.91$ cm

$$h = \frac{375}{\pi \times 3.90796^2} = 7.82\ cm. \ ✓$$

+ve

OR $(S.A)'' = 37.70$ ∴ MIN

Question 3 **(10 marks)**

Let $f(x) = -(x+1)^2(x-3)$.

(a) Use calculus to locate and classify all the stationary points of $f(x)$ and find any points of inflection. (6 marks)

$f'(x) = -2(x+1)(x-3) - (x+1)^2 \cdot 1$

$= -(x+1)(2x-6+x+1)$

$= -(x+1)(3x-5) = 0$ ✓

$x = -1$ or $x = \frac{5}{3}$ ✓

x	$(-\infty, -1)$	-1	$(-1, \frac{5}{3})$	$\frac{5}{3}$	$(\frac{5}{3}, +\infty)$
$f'(x)$	-1	0	$+$	0	$-$
$f'(x)$	$\downarrow$	min	$\uparrow$	max	$\downarrow$

$f(-1) = 0$ local min $(-1, 0)$

$f(\frac{5}{3}) = \frac{256}{27}$ local max $(\frac{5}{3}, \frac{256}{27})$ $\approx (9.48)$

point of inflection:

$f''(x) = -(3x-5) - (x+1) \times 3$

$= -3x + 5 - 3x - 3$

$= -6x + 2 = 0$ ✓

$x = \frac{1}{3}$, $y = -(\frac{1}{3}+1)^2(\frac{1}{3}-3)$

$= \frac{128}{27}$ (≈ 4.74)

P.O.I $(\frac{1}{3}, \frac{128}{27})$ ✓

OR

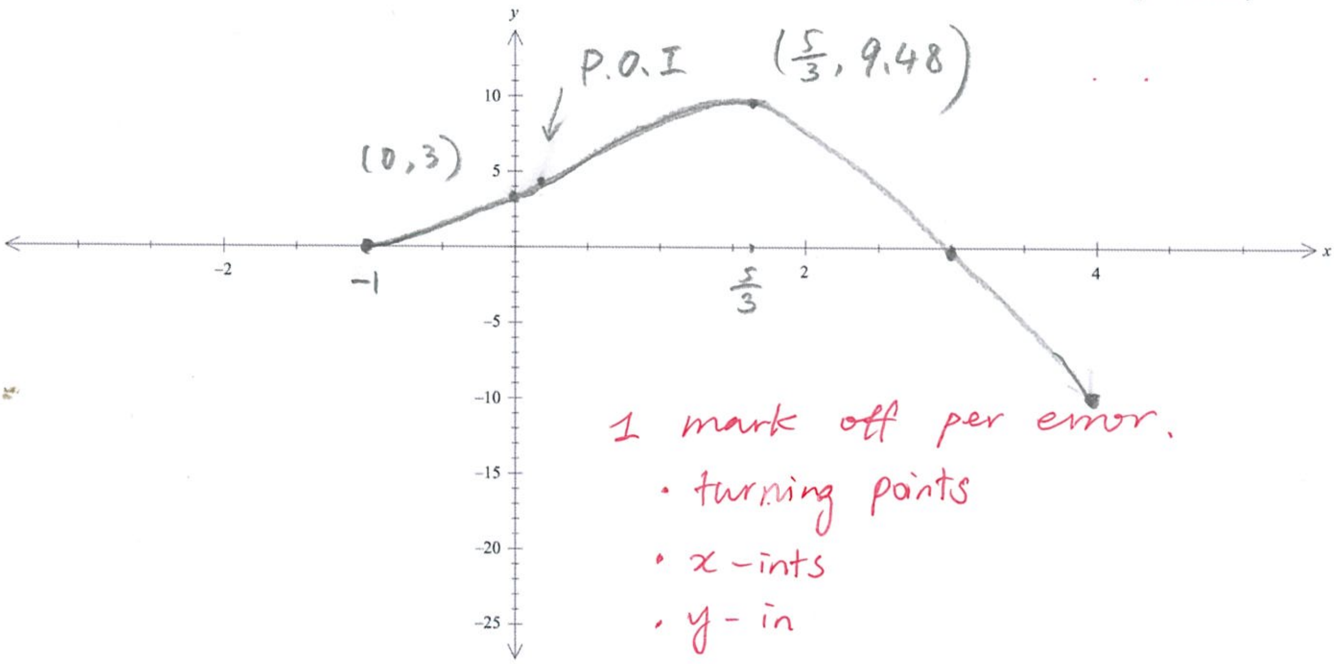
$f''(-1) = -6(-1) + 2 > 0$ ✓

$\therefore f(-1) = 0$, $(-1, 0)$ local min

$f''(\frac{5}{3}) = -(3 \times \frac{5}{3} - 5) - (\frac{5}{3} + 1) \times 3 < 0$

$\therefore f(\frac{5}{3}) = \frac{256}{27}$, $(\frac{5}{3}, \frac{256}{27})$ local max. ✓

(b) On the axes provided sketch the graph of $f(x)$, $-1 \le x \le 4$, labelling all key features. (4 marks)



1 mark off per error.

- turning points
- x-ints
- y-in
- P.O.I.
- shape
- domain